# ON AN INTENSITY-RATIO EQUIVALENCE-THEOREM

# FOR TOP QUARK DECAY

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## Abstract

For the $t \to W^+ b$ decay mode, an intensity-ratio equivalence-theorem for two Lorentz-invariant couplings is shown to be related to symmetries of tWb-transformations. Explicit tWb-transformations, $A_+ = M \, A_{SM}, \, P \, A_{SM}, \, B \, A_{SM}$ relate the four standard model's helicity amplitudes, $A_{SM} (\lambda_{W^+}, \lambda_b)$, and the amplitudes $A_+ (\lambda_{W^+}, \lambda_b)$ in the case of an additional $t_R \to b_L$ weak-moment of relative strength $\Lambda_+ = E_W/2 \sim 53 GeV$. Two commutator plus anti-commutator symmetry algebras are generated from $M, P, B$. These transformations enable a characterization of the associated mass scales.

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# 1 Introduction:

In this paper, for the $t \rightarrow W^+ b$ decay mode [1], an intensity-ratio equivalence-theorem [2] for two Lorentz-invariant couplings is shown to be related to symmetries of tWb-transformations, $A_+ = M \ A_{SM}, \ P \ A_{SM}, \ B \ A_{SM}$, where $M, P, B$ are explicit 4x4 matrices. These tWb-transformations relate the standard model's helicity amplitudes, $A_{SM} (\lambda_{W^+}, \lambda_b)$, and the amplitudes $A_+ (\lambda_{W^+}, \lambda_b)$ in the case of an additional $t_R \rightarrow b_L$ weak-moment of relative strength $\Lambda_+ = E_W/2 \sim 53 GeV$. Versus the standard model's pure $(V-A)$ coupling, the additional tensorial coupling can be physically interpreted as arising due to a large chiral weak-transition moment for the top quark. $\Lambda_+$ is defined by (1) below and the $(+)$ amplitudes' complete coupling is (2). $E_W$ is the energy of the final W-boson in the decaying top-quark rest frame. The subscripts $R$ and $L$ respectively denote right and left chirality of the coupling, that is $(1 \pm \gamma_5)$. $\lambda_{W^+}$, $\lambda_b$ are the helicities of the the emitted W-boson and b-quark in the top-quark rest frame. The Jacob-Wick phase-convention [3] is used in specifying the phases of the helicity amplitudes and so of these transformations.

Due to rotational invariance, there are four independent $A (\lambda_{W^+}, \lambda_b)$ amplitudes for the most general Lorentz coupling. Stage-two spin-correlation functions were derived and studied as a basis for complete measurements of the helicity parameters for $t \rightarrow W^+ b$ decay as tests with respect to the most general Lorentz coupling [4,5]. Such tests are possible at the Tevatron [1], at the LHC [6], and at a NLC [7]. In this paper, a subset of the most general Lorentz coupling is considered in which the subscript "$i$" identifies the amplitude's associated coupling: "$i = $ SM" for the pure $(V - A)$ coupling, "$i = (f_M + f_E)$" for only the additional $t_R \rightarrow b_L$ tensorial coupling, and "$i = (+)$" for $(V - A) + (f_M + f_E)$ with a top-quark chiral weak-transition moment of

relative strength $\Lambda_+ = E_W/2$ versus $g_L$. The Lorentz coupling involving both the SM's $(V - A)$ coupling and an additional $t_R \to b_L$ weak-moment coupling of arbitrary relative strength $\Lambda_+$ is $W_\mu^* J_{bt}^\mu = W_\mu^* \bar{u}_b(p) \Gamma^\mu u_t(k)$ where $k_t = q_W + p_b$, and

$$\frac{1}{2}\Gamma^\mu = g_L \gamma^\mu P_L + \frac{g_{f_M + f_E}}{2\Lambda_+} \iota\sigma^{\mu\nu}(k - p)_\nu P_R \qquad (1)$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$. In $g_L = g_{f_M + f_E} = 1$ units, when $\Lambda_+ = E_W/2$ which corresponds to the $(+)$ amplitudes, the complete $t \to b$ coupling is very simple

$$\gamma^\mu P_L + \iota\sigma^{\mu\nu} v_\nu P_R = P_R \left(\gamma^\mu + \iota\sigma^{\mu\nu} v_\nu\right) \qquad (2)$$

where $v_\nu$ is the W-boson's relativistic four-velocity.

In the $t$ rest frame, the helicity-amplitude matrix element for $t \to W^+ b$ is $\langle \theta_1^t, \phi_1^t, \lambda_{W^+}, \lambda_b | \frac{1}{2}, \lambda_1 \rangle = D_{\lambda_1, \mu}^{(1/2)*}(\phi_1^t, \theta_1^t, 0) A_i(\lambda_{W^+}, \lambda_b)$ where $\mu = \lambda_{W^+} - \lambda_b$ in terms of the $W^+$ and $b$-quark helicities. The asterisk denotes complex conjugation, the final $W^+$ momentum is in the $\theta_1^t, \phi_1^t$ direction, and $\lambda_1$ gives the $t$-quark's spin component quantized along the $z$ axis. $\lambda_1$ is also the helicity of the $t$-quark if one has boosted, along the "$-z$" direction, back to the $t$ rest frame from the $(t\bar{t})_{cm}$ frame. It is this boost which defines the $z$ axis in the $t$-quark rest frame for angular analysis [4]. Explicit expressions for the helicity amplitudes associated with each "$i$" coupling are listed in Sec. 2. We denote by $\Gamma$ the partial-width for the $t \to W^+ b$ decay channel and by $\Gamma_{L,T}$ the partial-width's for the sub-channels in which the $W^+$ is respectively longitudinally, transversely polarized; $\Gamma = \Gamma_L + \Gamma_T$. Similarly, $\Gamma_L|_{\lambda_b = -\frac{1}{2}}$ denotes the partial-width for the W-longitudinal sub-channel with b-quark helicity $\lambda_b = -\frac{1}{2}$.

The intensity-ratio equivalence-theorem states, "As consequence of Lorentz-invariance, for the $t \to W^+ b$ decay channel each of the four ratios $\Gamma_L|_{\lambda_b = -\frac{1}{2}}/\Gamma$, $\Gamma_T|_{\lambda_b = -\frac{1}{2}}/\Gamma$, $\Gamma_L|_{\lambda_b = \frac{1}{2}}/\Gamma$, $\Gamma_T|_{\lambda_b = \frac{1}{2}}/\Gamma$, is

identical for the pure $(V-A)$ coupling and for the $(V-A)+(f_M+f_E)$ coupling with $\Lambda_+ = E_W/2$, and their respective partial-widths are related by $\Gamma_+ = v^2\Gamma_{SM}$. $v$ is the velocity of the W-boson in the t-quark rest frame." Note that this theorem does not require specific values of the mass ratios $y \equiv m_W/m_t$, and $x \equiv m_b/m_t$, but that by setting $\Lambda_+ = E_W/2$ the relative strength of the chiral weak-transition moment for the top quark has been fixed versus $g_L$.

The three tWb-transformations, $A_+ = M\ A_{SM}$, $P\ A_{SM}$, $B\ A_{SM}$, are related to this theorem. The $M$ transformation implies the theorem, but as explained below, $M$ also implies the sign and ratio differences of the (ii) and (iii) type amplitude ratio-relations which distinguish the (SM) and (+) couplings. The $P$ and $B$ transformations more completely exhibit the underlying symmetries relating these two Lorentz-invariant couplings. In particular, these three 4x4 matrices lead to two "commutator plus anti-commutator" symmetry algebras, and together enable a characterization of the values of $\Lambda_+$, $y \equiv m_W/m_t$, and $x \equiv m_b/m_t$. In Sec. 2, it is shown how these three tWb-transformations successively arise from consideration of different types of "helicity amplitude relations" for $t \to W^+b$ decay: The type (i) are ratio-relations which hold separately for the two cases, "$i = (SM), (+)$". The type (ii) are ratio-relations which relate the amplitudes in the two cases. By the type (iii) ratio-relations, the tWb-transformation $A_+ = M\ A_{SM}$ where $M = v$ $diag(1, -1, -1, 1)$ characterizes the mass scale $\Lambda_+ = E_W/2$. Similarly, the amplitude condition (iv)

$$A_+(0, -1/2) = aA_{SM}(-1, -1/2), \tag{3}$$

with $a = 1 + O(v \neq y\sqrt{2}, x)$, determines the scale of the tWb-transformation matrix $P$ and determines the value of the mass ratio $y \equiv m_W/m_t$. $O(v \neq y\sqrt{2}, x)$ denotes small corrections, see

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below. The amplitude condition (v)

$$A_+(0, -1/2) = -bA_{SM}(1 - 1/2), \tag{4}$$

with $b = v^{-8}$, determines the scale of $B$ and determines the value of $x = m_b/m_t$. In Sec. 3, the two symmetry algebras are obtained which involve the $M$, $P$, and $B$ transformation matrices. Sec. 4 contains a discussion of these results and their implications assuming that the observed $t \to W^+ b$ decay mode will be found empirically to be well-described by (2).

## 2 Helicity amplitude relations:

In the Jacob-Wick phase convention, the helicity amplitudes for the most general Lorentz coupling are given in [4]. In $g_L = g_{f_M + f_E} = 1$ units and suppressing a common overall factor of $\sqrt{m_t (E_b + q_W)}$, for only the $(V - A)$ coupling the associated helicity amplitudes are:

$$A_{SM}\left(0, -\frac{1}{2}\right) = \frac{1}{y}\frac{E_W + q_W}{m_t}$$

$$A_{SM}\left(-1, -\frac{1}{2}\right) = \sqrt{2}$$

$$A_{SM}\left(0, \frac{1}{2}\right) = -\frac{1}{y}\frac{E_W - q_W}{m_t}\left(\frac{m_b}{m_t - E_W + q_W}\right)$$

$$A_{SM}\left(1, \frac{1}{2}\right) = -\sqrt{2}\left(\frac{m_b}{m_t - E_W + q_W}\right)$$

For only the $(f_M + f_E)$ coupling, i.e. only the additional $t_R \to b_L$ tensorial coupling:

$$A_{f_M + f_E}\left(0, -\frac{1}{2}\right) = -(\frac{m_t}{2\Lambda_+})\, y$$

$$A_{f_M + f_E}\left(-1, -\frac{1}{2}\right) = -(\frac{m_t}{2\Lambda_+})\sqrt{2}\,\frac{E_W + q_W}{m_t}$$

$$A_{f_M + f_E}\left(0, \frac{1}{2}\right) = (\frac{m_t}{2\Lambda_+})y\left(\frac{m_b}{m_t - E_W + q_W}\right)$$

$$A_{f_M + f_E}\left(1, \frac{1}{2}\right) = (\frac{m_t}{2\Lambda_+})\sqrt{2}\,\frac{E_W - q_W}{m_t}\left(\frac{m_b}{m_t - E_W + q_W}\right)$$

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From these, the amplitudes for the $(V - A) + (f_M + f_E)$ coupling of (1) are obtained by $A_+(\lambda_W, \lambda_b) = A_{SM}(\lambda_W, \lambda_b) + A_{f_M + f_E}(\lambda_W, \lambda_b)$. For $\Lambda_+ = E_W/2$, the $A_+(\lambda_W, \lambda_b)$ amplitudes corresponding to the complete $t \to b$ coupling (2) are

$$
\begin{aligned}
A_+\left(0, -\frac{1}{2}\right) &= \frac{1}{y}\,(q/E_W)\,\frac{E_W + q_W}{m_t} \\
A_+\left(-1, -\frac{1}{2}\right) &= -\sqrt{2}\,(q/E_W) \\
A_+\left(0, \frac{1}{2}\right) &= \frac{1}{y}\,(q/E_W)\,\frac{E_W - q_W}{m_t}\left(\frac{m_b}{m_t - E_W + q_W}\right) \\
A_+\left(1, \frac{1}{2}\right) &= -\sqrt{2}\,(q/E_W)\left(\frac{m_b}{m_t - E_W + q_W}\right)
\end{aligned}
$$

For the three "$i$" couplings, a direct derivation from (1) shows how the different factors arise in the amplitudes [8].

We now analyze the different types of helicity amplitude relations involving both the SM's amplitudes and those in the case of the $(V - A) + (f_M + f_E)$ coupling: The first type of ratio-relations holds separately for $i = (SM), (+)$ and for all $y = \frac{m_W}{m_t}, x = \frac{m_b}{m_t}, \Lambda_+$ values, (i):

$$
\frac{A_i(0, 1/2)}{A_i(-1, -1/2)} = \frac{1}{2}\frac{A_i(1, 1/2)}{A_i(0, -1/2)} \tag{5}
$$

The second type of ratio-relations relates the amplitudes in the two cases and also holds for all $y, x, \Lambda_+$ values. The first two relations have numerators with opposite signs and denominators with opposite signs, c.f. Table 1; (ii): Two sign-flip relations

$$
\frac{A_+(0, 1/2)}{A_+(-1, -1/2)} = \frac{A_{SM}(0, 1/2)}{A_{SM}(-1, -1/2)} \tag{6}
$$

$$
\frac{A_+(0, 1/2)}{A_+(-1, -1/2)} = \frac{1}{2}\frac{A_{SM}(1, 1/2)}{A_{SM}(0, -1/2)} \tag{7}
$$

and two non-sign-flip relations

$$
\frac{A_+(1, 1/2)}{A_+(0, -1/2)} = \frac{A_{SM}(1, 1/2)}{A_{SM}(0, -1/2)} \tag{8}
$$

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$$\frac{A_+(1, 1/2)}{A_+(0, -1/2)} = 2\frac{A_{SM}(0, 1/2)}{A_{SM}(-1, -1/2)} \tag{9}$$

(7, 9), which are not in [2], are essential for obtaining the $P$ and $B$ tWb-transformations and thereby the symmetry algebras of Sec. 3 below.

The third type of ratio-relations, holding for all $y, x$ values, follows by determining the effective mass scale, $\Lambda_+$, so that there is an exact equality for the ratio of left-handed amplitudes (iii):

$$\frac{A_+(0, -1/2)}{A_+(-1, -1/2)} = -\frac{A_{SM}(0, -1/2)}{A_{SM}(-1, -1/2)}, \tag{10}$$

Equivalently, $\Lambda_+ = \frac{m_t}{4}[1 + (m_W/m_t)^2 - (m_b/m_t)^2] = E_W/2$ follows from each of:

$$\frac{A_+(0, -1/2)}{A_+(-1, -1/2)} = -\frac{1}{2}\frac{A_{SM}(1, 1/2)}{A_{SM}(0, 1/2)}, \tag{11}$$

$$\frac{A_+(0, 1/2)}{A_+(1, 1/2)} = -\frac{A_{SM}(0, 1/2)}{A_{SM}(1, 1/2)}, \tag{12}$$

$$\frac{A_+(0, 1/2)}{A_+(1, 1/2)} = -\frac{1}{2}\frac{A_{SM}(-1, -1/2)}{A_{SM}(0, -1/2)}, \tag{13}$$

From the amplitude expressions given above, the value of this scale $\Lambda_+$ can be characterized by postulating the existence of a tWb-transformation $A_+ = M\, A_{SM}$ where $M = v\, diag(1, -1, -1, 1)$, with $A_{SM} = [A_{SM}(0, -1/2), A_{SM}(-1, -1/2), A_{SM}(0, 1/2), A_{SM}(1, 1/2)]$ and analogously for $A_+$.

Assuming (iii), the fourth type of relation is the equality (iv):

$$A_+(0, -1/2) = aA_{SM}(-1, -1/2), \tag{14}$$

where $a = 1 + O(v \neq y\sqrt{2}, x)$.

This is equivalent to the velocity formula $v = ay\sqrt{2}\left(\frac{1}{1-(E_b-q_W)/m_t}\right) = ay\sqrt{2}$ for $m_b = 0$. In [2], for $a = 1$ it was shown that (iv) leads to a mass relation with the solution $y = \frac{m_W}{m_t} = 0.46006$ ($x = 0$). The present empirical value is $y = 0.461 \pm 0.014$, where the error is dominated by the

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3% precision of $m_t$. In [2], for $a = 1$ it was also shown that (iv) leads to $\sqrt{2} = v\gamma(1+v) = v\sqrt{\frac{1+v}{1-v}}$ so $v = 0.6506\ldots$ without input of a specific value for $m_b$. However, by Lorentz invariance $v$ must depend on $m_b$. Accepting (iii), we interpret this to mean that $a \neq 1$ and in the Appendix obtain the form of the $O(v \neq y\sqrt{2}, x)$ corrections in $a$ as required by Lorentz invariance. The small correction $O(v \neq y\sqrt{2}, x)$ depends on both $x \equiv m_b/m_t$ and the difference $v - y\sqrt{2}$.

Equivalently, by use of (i)-(iii) relations, (14) can be expressed postulating the existence of a second tWb-transformation $A_+ = P\, A_{SM}$ where

$$P \equiv v \begin{bmatrix} 0 & a/v & 0 & 0 \\ -v/a & 0 & 0 & 0 \\ 0 & 0 & 0 & -v/2a \\ 0 & 0 & 2a/v & 0 \end{bmatrix} \tag{15}$$

The value of the parameter $a$ of (iv) is not fixed by (15).

The above two tWb-transformations do not relate the $\lambda_b = -\frac{1}{2}$ amplitudes with the $\lambda_b = \frac{1}{2}$ amplitudes. From (i) thru (iv), in terms of a parameter $b$, the equality (v):

$$A_+(0, -1/2) = -bA_{SM}(1, 1/2), \tag{16}$$

is equivalent to $A_+ = B\, A_{SM}$

$$B \equiv v \begin{bmatrix} 0 & 0 & 0 & -b/v \\ 0 & 0 & 2b/v & 0 \\ 0 & v/2b & 0 & 0 \\ -v/b & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

The choice of $b = v^{-8} = 31.152$, gives

$$B \equiv v \begin{bmatrix} 0 & 0 & 0 & -v^{-9} \\ 0 & 0 & 2v^{-9} & 0 \\ 0 & v^9/2 & 0 & 0 \\ -v^9 & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

and corresponds to the mass relation $m_b = \frac{m_t}{b}\left[1 - \frac{vy}{\sqrt{2}}\right] = 4.407...GeV$ for $m_t = 174.3GeV$.

# 3 Commutator plus anti-commutator symmetry algebras:

The anti-commuting matrices

$$m \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, p \equiv \begin{bmatrix} 0 & -a/v \\ v/a & 0 \end{bmatrix}, q \equiv \begin{bmatrix} 0 & a/v \\ v/a & 0 \end{bmatrix} \tag{19}$$

satisfy $[m, p] = -2q, [m, q] = -2p, [p, q] = -2m$. Similarly, $m$ and

$$r \equiv \begin{bmatrix} 0 & -v/2a \\ 2a/v & 0 \end{bmatrix}, s \equiv \begin{bmatrix} 0 & v/2a \\ 2a/v & 0 \end{bmatrix} \tag{20}$$

are anti-commuting and satisfy $[m, r] = -2s, [m, s] = -2r, [r, s] = -2m$. Note $m^2 = q^2 = s^2 = 1$, $p^2 = r^2 = -1$, and that $a$ is arbitrary. Consequently, if one does not distinguish the (+) versus SM indices, respectively of the rows and columns, the tWb-transformation matrices have some simple properties:

The anticommuting 4x4 matrices

$$M \equiv v \begin{bmatrix} m & 0 \\ 0 & -m \end{bmatrix}, P \equiv v \begin{bmatrix} -p & 0 \\ 0 & r \end{bmatrix}, Q \equiv v \begin{bmatrix} q & 0 \\ 0 & s \end{bmatrix} \tag{21}$$

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satisfy the closed algebra $[\overline{M},\overline{P}] = 2\overline{Q}, [\overline{M},\overline{Q}] = 2\overline{P}, [\overline{P},\overline{Q}] = 2\overline{M}$. The bar denotes removal of

the overall "$v$" factor, $M = v\overline{M}, ....$ Note that $Q$ is not a tWb-transformation.

Including the B matrix with $b$ arbitrary, the algebra closes with 3 additional matrices

$$\overline{B} \equiv \begin{bmatrix} 0 & d \\ f & 0 \end{bmatrix}, \overline{C} \equiv \begin{bmatrix} 0 & e \\ g & 0 \end{bmatrix} \tag{22}$$

$$\overline{G} \equiv \begin{bmatrix} 0 & h \\ k & 0 \end{bmatrix}, \overline{H} \equiv \begin{bmatrix} 0 & j \\ l & 0 \end{bmatrix} \tag{23}$$

where

$$d \equiv \begin{bmatrix} 0 & -b/v \\ 2b/v & 0 \end{bmatrix}, e \equiv \begin{bmatrix} 0 & b/v \\ 2b/v & 0 \end{bmatrix}, f \equiv \begin{bmatrix} 0 & v/2b \\ -v/b & 0 \end{bmatrix}, g \equiv \begin{bmatrix} 0 & v/2b \\ v/b & 0 \end{bmatrix} \tag{24}$$

$$h \equiv \begin{bmatrix} -2ab/v^2 & 0 \\ 0 & b/a \end{bmatrix}, j \equiv \begin{bmatrix} 2ab/v^2 & 0 \\ 0 & b/a \end{bmatrix}, k \equiv \begin{bmatrix} 1/2v^2ab & 0 \\ 0 & -a/b \end{bmatrix}, l \equiv \begin{bmatrix} 1/2v^2ab & 0 \\ 0 & a/b \end{bmatrix} \tag{25}$$

The squares of the 2x2 matrices (24-25) do depend on $a$, $b$, and $v$.

The associated closed algebra is: $[\overline{M},\overline{B}] = 0, \{\overline{M},\overline{B}\} = -2\overline{C}; [\overline{B},\overline{C}] = 0, \{\overline{B},\overline{C}\} = -2\overline{M};$

$[\overline{M},\overline{C}] = 0, \{\overline{M},\overline{C}\} = -2\overline{B};$ and $[\overline{P},\overline{B}] = 2\overline{H}, \{\overline{P},\overline{B}\} = 0; [\overline{H},\overline{P}] = 2\overline{B}, \{\overline{H},\overline{P}\} = 0;$

$[\overline{H},\overline{B}] = 2\overline{P}, \{\overline{H},\overline{B}\} = 0$ . Similarly, $[\overline{P},\overline{C}] = 0, \{\overline{P},\overline{C}\} = -2\overline{G}; [\overline{M},\overline{H}] = -2\overline{G},$

$\{\overline{M},\overline{H}\} = 0; [\overline{H},\overline{C}] = 0, \{\overline{H},\overline{C}\} = 2\overline{Q};$ and $[\overline{M},\overline{G}] = -2\overline{H}, \{\overline{M},\overline{G}\} = 0; [\overline{P},\overline{G}] = 0,$

$\{\overline{P},\overline{G}\} = 2\overline{C}; [\overline{G},\overline{B}] = -2\overline{Q}, \{\overline{G},\overline{B}\} = 0;$ and $[\overline{G},\overline{C}] = 0, \{\overline{G},\overline{C}\} = -2\overline{P};$

$[\overline{G},\overline{H}] = 2\overline{M}, \{\overline{G},\overline{H}\} = 0$. The part involving $\overline{Q}$ is $[\overline{G},\overline{Q}] = 2\overline{B}, \{\overline{G},\overline{Q}\} = 0; [\overline{B},\overline{Q}] = 2\overline{G},$

$\{\overline{B},\overline{Q}\} = 0; [\overline{C},\overline{Q}] = 0, \{\overline{C},\overline{Q}\} = -2\overline{H}; [\overline{H},\overline{Q}] = 0, \{\overline{H},\overline{Q}\} = 2\overline{C}.$

This has generated an additional tWb-transformation $G \equiv v\overline{G}$; but $C \equiv v\overline{C}$ and $H \equiv v\overline{H}$ are not tWb-transformations.

Up to the insertion of an overall $\iota = \sqrt{-1}$, each of these 4x4 barred matrices is a resolution of unity, i.e. $\overline{P}^{-1} = -\overline{P}$, $\overline{G}^{-1} = -\overline{G}$, but $\overline{Q}^{-1} = \overline{Q}$, $\overline{B}^{-1} = \overline{B}$, ... .

# 4   Discussion:

**(1) $\Lambda_+$ mass scale:**

A fundamental question [2] raised by the existence of the intensity-ratio equivalence theorem is "What is the origin of the $\Lambda_+ = E_W/2 \sim 53 GeV$ mass scale?" The present paper shows that $M$ is but one of three logically-successive tWb-transformations which are constrained by the helicity amplitude ratio-relations (i) and (ii). Thereby, the type (iii) ratio-relation fixes $\Lambda_+ = E_W/2$ and the overall scale of the tWb-transformation matrix $M$. The amplitude condition (iv), $A_+(0, -1/2) = aA_{SM}(-1, -1/2)$ with $a = 1 + O(v \neq y\sqrt{2}, x)$, and the amplitude condition (v), $A_+(0, -1/2) = -bA_{SM}(1 - 1/2)$ with $b = v^{-8}$, determine respectively the scale of the tWb-transformation matrices $P$ and $B$ and characterize the values of $m_W/m_t$ and $m_b/m_t$. The overall scale can be set here by $m_t$ or $m_W$. From an empirical "bottom-up" perspective of further "unification", $m_W$ is more appropriate to use to set these scales since its value is fixed in the SM by the vacuum expectation value of the Higgs field, $\phi$ [ the SM's EW scale is $v_{EW} = \sqrt{-\mu^2/|\lambda|} = \sqrt{2}\langle 0|\phi|0\rangle \sim 246 GeV$]. So by postulating the $M$ transformation, the $\Lambda_+$ mass scale is fixed as $E_W/2$. By these three tWb-transformations, the numerical value of $\Lambda_+$ is determined by that of $v_{EW}$.

**(2) Comparison with an Amplitude Equivalence-Theorem:**

Given the continued successes of predictions based on the couplings and symmetries built into the SM, and given the present rather slow pace of new experimental information, we appreciate the fact that for many readers it can be difficult to remind oneself that directly from experiment we still really do not know much about the properties of the on-shell top-quark [1]. Because of possible form factor effects and possible unknown thresholds due to new particles, in a theory/model-independent manner one cannot reliably determine the indirect constraints on on-shell top-quark couplings from off-shell contributions from top-quark contributions in higher-order loops in electroweak precision tests [2]. Various assumptions of quark universality are still routinely made in the theoretical literature to conjecture on-shell top-quark properties from theoretical patterns found for the several-order-of-magnitude less massive quarks, in spite of the closeness of the value of $m_t$ to $v_{EW}$ and of the now significantly greater numerical-precision (3%) of the $m_t$ measurement [1] than that of any of the other quark masses.

This present "not-knowing" status quo is very different from that in 1940-1952 in regard to the then rapidly changing and developing experimental status quo in the case of the weak and strong interactions, which was concurrent with the series of striking empirical-theoretical successes with QED ( QED is often viewed as the prototypical earlier analogue of the present SM ). Nevertheless, despite these differences in the experimental situation, we think it is instructive to compare this present intensity-ratio equivalence theorem with a somewhat analogous "amplitude equivalence theorem (ET)" which was discovered and quite intensely studied, circa 1940-1952, for the pseudoscalar and pseudovector interactions, Dyson (1948) [9].

Although not as influential as Fermi's 1934 paper and the Gamow-Teller 1936 paper concerning

the Lorentz structure of the weak interactions, this ET has had a long, fruitful, and significant impact in high energy experiment and in theoretical elementary particle physics. The early history and early significance of the ET , e.g. [9] and [10], can be traced from Schweber's book (1962) [11] and from the entire final chapter of Schweber, Bethe, and de Hoffmann, (1955) [12]. This ET stimulated in part, the development of effective Lagrangian methods [13] and work by G. t'Hooft and M. Veltman [14]. ( Later ET literature cites the t'Hooft-Veltman preprint; the corresponding published tHV-paper does not cite, e.g. [12], but refers to their tHV-paper as being based on unpublished preprints. ) The "theorem" continues to be of theoretical interest, see for instance [15]. In top-quark physics, we think it is reasonable to expect that the theoretical patterns, analytic relationships and tWb-transformations developed in the present paper between the helicity amplitudes for this specific additional tensorial coupling and those for the $(V - A)$ coupling will have such an early experimental-theoretical history.

There are other similarities and other important qualitative differences between the present situation and that concerning the ET. They are similar in that both relate simple Lorentz structures, involve helicities...though more intricately in the present case, and involve a renormalizable coupling. Major differences include (I) the ET case is much better understood after over 60 years of research papers, whereas this is only a 2nd paper towards understanding the patterns and possible physics of this aspect of top quark decays, (II) the SM is now known to well explain most of the weak interaction systems (nucleon, nucleus, strange particle weak decays) first studied by the ET stimulating experiments, whereas experiments have only begun on top quark decay, and (III) the tWb transformations involve couplings of a fundamental renormalizable local quantum field theory, the SM, and fundamental mass ratios, whereas we now know that the ET case never did.

**(3) Possible Implications of These Symmetries:**

The additional $t_R \rightarrow b_L$ weak-moment coupling violates the conventional gauge invariance transformations of the SM and traditionally in electroweak studies such anomalous couplings have been best considered as "induced" or "effective". Nevertheless, in special "new physics" circumstances such a simple charge-changing tensorial coupling as (2) might turn out to be a promising route to deeper understandings. The "tensorial coupling" is a basic structure if considered from gravitation viewpoints. However, are the new symmetries associated with the symmetry algebras of Sec. 3 sufficient to overcome the known difficulties [16] in constructing a renormalizable, unitary quantum field theory involving second class currents [17] ? The $f_E$ component is second class. $f_E$ has a distinctively different reality structure, and time-reversal invariance property versus the first class $V, A, f_M$ [18]. If the observed $t \rightarrow W^+b$ decay mode is found empirically to be well described by (2) this would support a working hypothesis that in the renormalization of the $t \rightarrow b$ coupling and of the top and bottom quark masses, the underlying symmetries of these tWb-transformations are basic to relating the associated mass scales, much as are Lorentz invariance and the symmetries of gauge-invariance-dynamics in performing renormalizations in the SM.

**(4) Experimental Tests/Measurements:**

In on-going [1] and forth-coming [6,7] top-quark decay experiments, important information about the relationship of the tWb-transformation symmetry patterns of this paper to the observed top quark decays will come from:

(a) Measurement of the sign of the $\eta_L \equiv \frac{1}{\Gamma}|A(-1,-\frac{1}{2})||A(0,-\frac{1}{2})|\cos\beta_L = \pm 0.46(SM/+)$ helicity parameter [4] so as to determine the sign of $cos\beta_L$ where $\beta_L = \phi^L_{-1} - \phi^L_0$ is the relative phase of the two $\lambda_b = -\frac{1}{2}$ amplitudes, $A(\lambda_{W^+}, \lambda_b) = |A|\exp(i\phi^{L,R}_{\lambda_{W^+}})$. For the exclusion of the coupling of (2)

versus the SM's $(V - A)$ coupling, this would be the definitive near-term measurement concerning properties of the on-shell top-quark.

(b) Measurement, or an empirical bound, for the closely associated

$\eta_L{}' \equiv \frac{1}{\Gamma}|A(-1, -\frac{1}{2})||A(0, -\frac{1}{2})| \sin \beta_L$ helicity parameter. This would provide useful complementary information, since in the absence of $T_{FS}$-violation, $\eta_L{}' = 0$ [4].

(c) Measurement of the partial width for $t \to W^+ b$ such as in single-top production [19-21]. The $v^2$ factor which differs their associated partial widths corresponds to the SM's $\Gamma_{SM} = 1.55 GeV$, versus $\Gamma_+ = 0.66 GeV$ and a longer-lived $(+)$ top-quark if this mode is dominant.

Since the helicity amplitude relations discussed in Sec. 2 involve the b-quark helicities, c.f. differing signs in $\lambda_b = 1/2$ column of Table 1, there are also independent phase tests which require

(d) Measurements of helicity parameters [5] using $\Lambda_b$-polarimetry in stage-two spin-correlation functions. It is noteworthy that the $\Lambda_b$ baryon has been observed by CDF at the Tevatron [22].

**Acknowledgments:**

**Appendix: The $O(v \neq y\sqrt{2}, x)$ corrections in $a$**

In this appendix is listed the form of the $O(v \neq y\sqrt{2}, x)$ corrections in $a$ as required by Lorentz invariance:

For $a = 1 + \varepsilon(x, y)$, the (iv) relation is $v = (1 + \varepsilon)y\sqrt{2}m_t/(E_W + q)$ whereas from relativistic kinematics $v = q/E_W = [(1 - y^2 - x^2)^2 - 4y^2 x^2]^{1/2}/[1 + y^2 - x^2]$. By equating these expressions and expanding in $x$, one obtains $\varepsilon = R + x^2 S$ where

$$R = \frac{1 - 4y^2 - 3y^4 - 2y^6}{4y^2(1 + y^2)^2}$$

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$$S = \frac{-1 - 4y^2 + y^4}{2y^2(1 + y^2)^3}$$

and $v = y\sqrt{2}\left[1 + R + x^2(S + \frac{1+R}{1-y^2}) + O(x^4)\right]$. From the latter equation, $R = (v - y\sqrt{2})/y\sqrt{2} + O(x^2)$.

For a massless b-quark ( $x = 0$ ) and $a = 1$ , the (iv) relation is equivalent to the $\frac{m_W}{m_t}$ mass relation $y^3\sqrt{2} + y^2 + y\sqrt{2} - 1 = 0$, and by relativistic kinematics to the W-boson velocity condition $v^3 + v^2 + 2v - 2 = 0$ and the simple formula $v = y\sqrt{2}$.

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Table 1: Numerical values of the helicity amplitudes for the standard model $(V - A)$ coupling and for the $(+)$ coupling of Eq.(2). The latter consists of an additional $t_R \to b_L$ weak-moment of relative strength $\Lambda_+ \sim 53 GeV$ so as yield a relative-sign change in the $\lambda_b = -\frac{1}{2}$ amplitudes. The values are listed first in $g_L = g_{f_M + f_E} = 1$ units, and second as $A_{new} = A_{g_L=1}/\sqrt{\Gamma}$. Table entries are for $m_t = 175 GeV$, $m_W = 80.35 GeV$, $m_b = 4.5 GeV$.

Table 1: Helicity amplitudes for $(V - A)$ coupling and the $(+)$ coupling of Eq.(2).

| | $A(0, -\frac{1}{2})$ | $A(-1, -\frac{1}{2})$ | $A(0, \frac{1}{2})$ | $A(1, \frac{1}{2})$ |
|---|---|---|---|---|
| $A_{g_L=1}$ in $g_L = 1$ units | | | | |
| $V - A$ | 338 | 220 | $-2.33$ | $-7.16$ |
| $f_M + f_E$ | 220 | $-143$ | 1.52 | $-4.67$ |
| $A_{New} = A_{g_L=1}/\sqrt{\Gamma}$ | | | | |
| $V - A$ | 0.84 | 0.54 | $-0.0058$ | $-0.018$ |
| $f_M + f_E$ | 0.84 | $-0.54$ | 0.0058 | $-0.018$ |